NUSSELT NUMBERS FOR A POWER-LAW FLUID FLOW BETWEEN FIXED PARALLEL PLATES WITH CONSTANT HEAT FLUXES IN THE PRESENCE OF VISCOUS DISSIPATION

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ABSTRACT

Fully developed laminar heat transfer of non-Newtonian fluids between fixed parallel plates has been studied both thermally and hydrodynamically, accounting for the effect of the moving fluid's viscous dissipation. Both plates are kept at distinct constant heat fluxes as the thermal boundary condition. Analytical solutions to the energy equation and subsequent Nusselt number were found using the Brinkman number and power-law index. The results demonstrate that the power-law index of the moving fluid has an impact on heat transmission. Pseudo-plastic and dilatant fluids exhibit differing heat transfer characteristics when viscous dissipation is present. It is important to take into account the major impacts of viscous dissipation on heat transmission between parallel plates under specific circumstances.

Keywords: Nusselt number, Power-law fluid flow, Brinkman number, Viscous dissipation, Constant heat flux.

Nomenclature

$A_1 - A_6$	coefficients defined in Equations. (18), (19), and (26)-(29)
A_C	cross-sectional area of channel (m²)
Br_{q_1}	modified Brinkman number defined in Eq. (7)
$C_1 - C_4$	coefficients defined in Equations. (12)-(15)
c_p	specific heat at constant pressure (J/kg K)
h	convective heat transfer coefficient (W/m²K)
k	thermal conductivity (W/m K) width of plate (m)
L	power-law index
Nu	Nusselt number, defined in Eq. (24)
P	pressure (Pa)

q_1	upper wall heat flux (W/m2)
q_2	lower wall heat flux (W/m²)
T	temperature (K)
T_0	wall temperature when both walls are kept at the same constant heat flux (K)
T_1	upper wall temperature (K)
T_2	lower wall temperature (K) AT
ςT	general temperature difference (K)
u	velocity (m/s)
U	dimensionless velocity
w	half-channel height (m)
W	channel height (=2w) (m)
x	coordinate in the axial direction (m)
y	coordinate in the vertical direction (m)
Y	dimensionless vertical coordinate
Greek symbols	
α	thermal diffusivity (m²/s)
β	parameter defined in Eq. (7)
γ	parameter defined in Eq. (9)
θ	dimensionless temperature
θ_m	mean dimensionless temperature
η	consistency factor (pas")
ρ	density (kg/m³)
τ	shear-stress (Pa)
Subscripts	
c	center-line
e	fluids entering
m	mean

1. Introduction

Heat transfer through viscous dissipation effects has been widely studied [1-5], and this effect commonly occurs in many applications such as material processing and high-velocity flows. However, the influence of viscous dissipation in non-Newtonian power law flow of liquids is relatively unknown. Considering various thermal boundary conditions, the simultaneous occurrence of steady laminar flow of a viscous non-Newtonian fluid flowing between parallel plates was investigated numerically [6]. When a fluid has a high viscosity, the flow is generally considered to be dynamically fully developed. This occurs in polymer processing where highly elastic fluids flow under non-isothermal conditions. The problem of heat intrusion in pipe and duct flow was modeled and solved semi-analytically by considering either a specified wall temperature or a specified wall shock flow as the thermal boundary condition [7] .

An analytical solution for viscoelastic fluids was obtained when one plate was exposed to a constant heat flow and the other plate was insulated but moved at a constant velocity, parallel plates [8]. To give meaning to viscous dissipation, a numerical study is performed on Poiseuille-Couette flow in a non-Newtonian fluid when one wall is exposed to a constant heat flow and the other wall is insulated [9]. Another work [10] addresses heat transfer due to the effects of viscous dissipation in the flow of non-Newtonian fluids through parallel plates and circular tubes with thermal boundary conditions of uniform wall temperature. In terms of the second law of thermodynamics, the effect of viscous dissipation in single-phase non-Newtonian fluids on entropy generation in circular microchannels was studied [11]. Considering a non-Newtonian fluid flowing in a channel of heated parallel plates and considering the effects of viscous dissipation, the second law was analyzed and the temperature and entropy generation was reported [12]. The effects of viscous dissipation and convective heat transfer in non-Newtonian thin liquid films on unstable stretch films have been discussed [13].

The analytical results for the case of both plates kept at different constant heat fluxes have not been documented in the literature, despite the fact that several investigations on the flow of power-law fluids with viscous dissipation in parallel plates have been conducted. As a result, the motivation behind the current analytical study is to carefully examine the modifications that the inclusion of the influence of viscous dissipation causes to the convection heat transfer characteristics for power-law fluids.

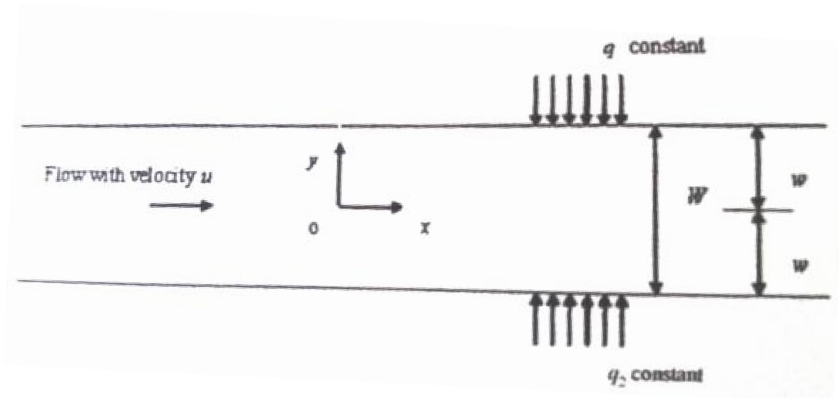

Fig. 1. Notation to the problem

2. Statement of the Problem and Mathematical Formulation

Considering a steady laminar flow of a non-Newtonian fluid with constant properties between fixed infinitely long parallel plates distanced W or $2w$ apart, to be fully developed both thermally and hydro-dynamically. The thermal boundary conditions, the case where the upper plate at constant heat flux q_1 while the lower plate at different constant heat flux q_2, as displayed in Fig. 1, is considered. The connection between shear-stress describes the rheological behavior of a power-law fluid between stationary parallel plates with constant fluid characteristics.

$$\tau = -\eta \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy}, \tag{1}$$

where r is the shear-stress, η is the consistency factor and du/dy is the velocity gradient perpendicular to the flow direction. The n is the power-law index, where the fluid is shear thinning or pseudo-plastic for $0 < n < 1$, Newtonian for $n = 1$ and shear thickening or dilatant for $n > 1$.

When the velocity boundary conditions are $u = 0$ when $y = w$ and $y = -w$, the maximum velocity, u_c, occurs midway $(y = 0)$ between the two parallel plates. Following this, the well-known velocity distribution is given by

$$u = u_c \left[1 - \left(\frac{y}{w} \right)^{(n+1)/n} \right] \tag{2}$$

The energy equation, including the effect of viscous dissipation, is given by

$$\rho C_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \eta \left| \frac{du}{dy} \right|^{n-1} \left(\frac{du}{dy} \right)^2, \tag{3}$$

where the second term on the right-hand side is the viscous-dissipative term. According to the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is ignored in the energy equation [14]. Therefore, the temperature gradient along the axial direction is independent of the transverse direction and is given by

$$\frac{\partial T}{\partial x} = \frac{dT_1}{dx} = \frac{dT_2}{dx} \tag{4}$$

where T_1 and T_2 are the upper and lower wall temperatures, respectively.

By taking $\alpha = k/\rho C_p$ and substituting Eqs. (2) and (4) into Eq. (3), it becomes

$$\frac{\partial^2 T}{\partial y^2} = \frac{u_c}{\alpha} \left[1 - \frac{y^{(n+1)/n}}{w^{(n+1)/n}} \right] \frac{dT_1}{dx} - \frac{\eta}{\alpha \rho c_p} u_c^{n+1} \left(\frac{n+1}{n} \right)^{n+1} \frac{y^{(n+1)/n}}{w^{(n+1)/n}} \tag{5}$$

By introducing the non-dimensional quantities

$$Y = \frac{y}{W}, \text{ and } \theta = \frac{T - T_1}{q_1 W / k}, \tag{6}$$

and by letting β . which is simply a dimensionless constant, and modified Brinkman number Br_{q_1}, respectively, be

$$\beta = \frac{u_c k W}{\alpha q_1}\frac{dT_1}{dx} \text{ and } Br_{q_1} = \frac{\eta u_c^{n+1}}{q_1 W^n}, \tag{7}$$

Eq. (5) can be written as

$$\frac{d^2\theta}{dY^2} = \beta - \gamma Y^{(n+1)/n}, \tag{8}$$

where

$$\gamma = 2^{(n+1)/n}\left\{\beta + \left[\frac{2(n+1)}{n}\right]^{n+1} Br_{q_1}\right\}, \tag{9}$$

The thermal boundary conditions are

$$k\frac{\partial T}{\partial y} = q_1 \text{ at } y = w, \text{ or } \frac{\partial \theta}{\partial Y} = 1 \text{ at } Y = \frac{1}{2}, \; T = T_1 \text{ at } y = w, \; \theta = 0 \text{ at } Y = \frac{1}{2} \tag{10}$$

The solution of Eq. (8) under the above thermal boundary conditions can be obtained as

$$\theta(Y) = C_1 Y^{(3n+1)/n} + C_2 Y^2 + C_3 Y + C_4, \tag{11}$$

where

$$C_1 = -\frac{\gamma n^2}{(2n+1)(3n+1)}, \tag{12}$$

$$C_2 = \frac{\beta}{2}, \tag{13}$$

$$C_3 = \frac{yn\left(\frac{1}{2}\right)^{(n+1)/n} + (2n+1)(2-\beta)}{2(2n+1)}, \tag{14}$$

$$C_4 = \frac{yn\left(\frac{1}{2}\right)^{(n+1)/n} + (3n+1)(4-\beta)}{8(3n+1)}, \tag{15}$$

To evaluate β in the above equation, a third boundary condition is required:

$$-k\frac{\partial T}{\partial y} = q_2 \text{ at } y = -w, \text{ or } \frac{\partial \theta}{\partial Y} = -\frac{q_2}{q_1} \text{ at } Y = -\frac{1}{2}, \tag{16}$$

By substituting Eq. (16) into Eq. (11), β can be expressed as

$$\beta = A_1\left(1 + \frac{q_2}{q_1}\right) + A_2 Br_{q_1}, \tag{17}$$

with the coefficients A_1, and A_2, in terms of n, defined as

$$A_1 = \frac{2n+1}{(2n+1) - 2^{(2n+1)/n}\, n\left[\left(\frac{1}{2}\right)^{(3n+1)/n} + \left(-\frac{1}{2}\right)^{(3n+1)/n}\right]} \tag{18}$$

$$A_2 = \frac{2^{\left(n^2+3n+1\right)/n}\, n\left[\left(n+1\right)/n\right]^{n+1}\left[\left(1/2\right)^{(3n+1)/n} + \left(-1/2\right)^{(3n+1)/n}\right]}{\left(2n+1\right) - 2^{(2n+1)/n}\, n\left[\left(1/2\right)^{(3n+1)/n} + \left(-1/2\right)^{(3n+1)/n}\right]} \tag{19}$$

In fully developed flow, it is usual to utilize the mean fluid- temperature, T_m . rather than the center-line temperature, when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m = \frac{\int_{A_c} \rho u T dA_c}{\int_{A_c} \rho u dA_c} \tag{20}$$

with A_c the cross-sectional area of the channel and the denominator on the right-hand side of Eq. (20) can be written as

$$\rho \int_{-w}^{w} u_c \left[1 - \left(\frac{y}{w}\right)^{(n+1)/n}\right] dA_c = \rho u_c L W \left(\frac{3n+2+n\left(-1\right)^{1/n}}{4n+2}\right) \tag{21}$$

Using Eqs. (2) and (11), the numerator of Eq. (20) can be found. The dimensionless mean temperature is therefore given by

$$\theta_m = \frac{k}{q_1 W}\left(T_m - T_1\right) \tag{22}$$

At this point, the convective heat transfer coefficient can be evaluated by

$$q_1 = h\left(T_1 - T_m\right) \tag{23}$$

Defining the Nusselt number to be

$$Nu = \frac{hW}{k} = \frac{q_1 W}{k\left(T_1 - T_m\right)} = -\frac{1}{\theta_m}, \tag{24}$$

the Nusselt number can be evaluated and its explicit expression be given as

$$Nu = \frac{A_3}{A_1 A_5\left(1 + {q_2}/{q_1}\right) + \left(A_4 + A_2 A_5\right) Br_{q_1} + A_6}, \tag{25}$$

where

$$A_3 = -12\left(5n+2\right)\left(4n+1\right)\left(3n+1\right)\left[3n+2+\left(-1\right)^{1/n}\right], \tag{26}$$

$$\begin{aligned} A_4 = 2^n\left(\frac{n+1}{n}\right)^n 6\Big[& n^2\left(4n+1\right)\left(n+1\right)+\left(-1\right)^{2/n}+n\left(35n^3+59n^2+28n+4\right)\left(-1\right)^{(n+1)/n} \\ & -\left(81n^4+17n^3+127n^2+38n+4\right)\Big], \end{aligned} \tag{27}$$

$$\begin{aligned} A_5 = {} & 6n\left(25n^3+35n^2+15n+2\right)\left(-1\right)^{1/n}+3n^3\left(4n+1\right)\left(-1\right)^{2/n}+222n^4+427n^3 \\ & +286n^2+80n+8, \end{aligned} \tag{28}$$

$$A_6 = 6n\left(100n^3+105n^2+36n+4\right)\left(-1\right)^{(n+1)/n} - 6\left(220n^4+343n^3+192n^2+46n+4\right), \tag{29}$$

3. Results and Discussion

Since the general results are too complex, various individual cases are presented below to illustrate the heat transfer characteristics. The values of n selected for discussion are $1/4$, $1/2$, 1, and 2.

3.1. Cases of Unequal Heat Fluxes

3.1.1. Newtonian Fluids

The heat transfer between the fluid and the upper wall is described by the Nusselt number in Eq. (24), which also takes into account the effect of viscous dissipation. For a Newtonian fluid $(n=1)$, we have the established result,

$$Nu = \frac{70}{26 - 9\left(q_2/q_1\right) + 24Br_{q_1}} \tag{30}$$

agreeing with Ref. [5].

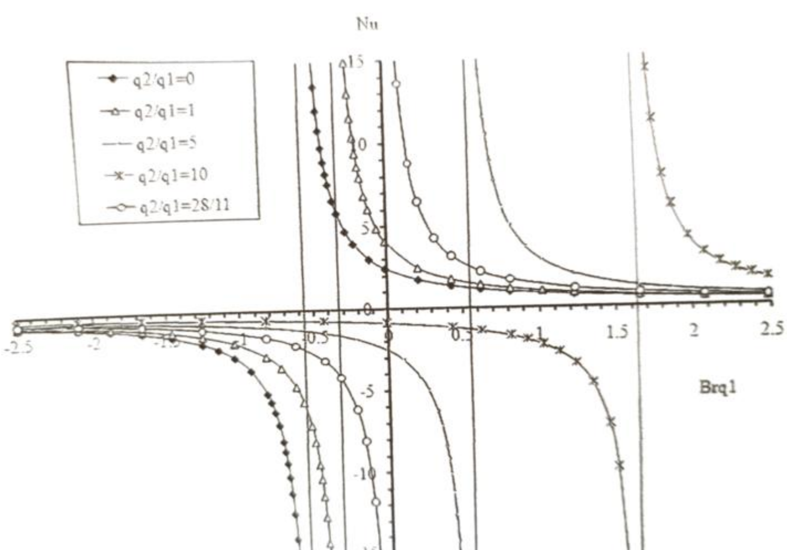

Fig. 2. Graph of Nu versus Br_{q_1} for $n = 1/4$

3.1.2. Shear Thinning Fluids

For the pseudo-plastic fluids $(n<1)$, when $n = 1/4$,

$$Nu = \frac{1}{16/39 - 44\left(q_2/q_1\right)/273 + 24711 Br_{q_1}/33721} \tag{31}$$

and when $n = 1/2$,

$$Nu = \frac{1}{4/9 - 7\left(q_2/q_1\right)/45 + 56638 Br_{q_1}/63061} \tag{32}$$

As expected, from Eq. (31), at a given ratio of $\left(q_2/q_1\right)$, the graph (Fig. 2) Nu at $n = 1/4$ versus Br_{q_1}, will form a rectangular hyperbola on both sides of an asymptote of

$$Br_{q_1} = -\left(\frac{539536}{963729}\right) + \frac{1483724\left(q_2/q_1\right)}{6746103} \tag{33}$$

Five sets of curves are shown in Fig. 2, for the heat flux ratios of $0, 1, 5, 10$ and $28/11$.

The ratio 0 corresponds to the case of insulated lower plate and the case of an equal constant heat flux on both plates is represented by the ratio of unity. The ratio $28/11$ is of interest because the asymptote lies on the vertical axis.

3.1.3. Shear Thickening Fluids

For dilatant fluids $(n>1)$, when $n=2$, the real part of Nu is

$$Nu = \frac{126\left[-320\left(q_2/q_1\right)+3753\,Br_{q_1}+976\right]}{6025\left(q_2/q_1\right)^2-368200\left(q_2/q_1\right)-142560\,Br_{q_1}\left(q_2/q_1\right)+463968\,Br_{q_1}+1102977\,Br_{q_1}^2+57028} \tag{34}$$

The $Nu-Br_{q_1}$, curves for dilatant fluids feature differently from those for Newtonian and pseudo-plastic fluids. Instead of manifesting as rectangular hyperbola with asymptotic values of Nu and Br_{q_1}, the $Nu-Br_{q_1}$, curves for dilatant fluids appear as non-asymptotic forms, showing turning points in the variation of Br_{q_1}, against Nu.

Table 1: Minimum and maximum points when Nu versus Br_{q_1}, for various ratios of $\left(q_2/q_1\right)$ at $n=2$.

q_2/q_1	Minimum points		Maximum points	
	Br_{q_1}	Nu	Br_{q_1}	Nu
0	-0.3598	-1.4346	-0.1604	4.2900
1	-0.2331	-2.4522	-0.1165	7.3339
2	-0.1065	-8.4395	-0.0726	25.240
2.25	-0.0748	-21.655	-0.0616	64.994
2.35	-0.0621	-57.933	-0.0572	172.72
2.45	-0.0528	-266.55	-0.0495	86.235
2.451	-0.0528	-250.25	-0.0494	83.500
2.455	-0.0526	-235.29	-0.0489	74.886
2.46	-0.0524	-215.12	-0.0482	67.759
2.48	-0.0515	-145.91	-0.0457	49.115
3	-0.0287	-17.500	0.0202	5.8529
4	0.0152	-6.4994	0.1468	2.1731
5	0.0591	-3.9904	0.2735	1.3344
10	0.2785	-1.3619	0.9067	0.4554

Table 1 displays the values of the turning points in the variation of Br_{q_1} , against Nusselt number for $n = 2$ for the specified heat flux ratios. Moreover, when ${q_2}/{q_1}$, increases from 0 to 2.45 the Nusselt number decreases as Br_{q_1} , increases. When ${q_2}/{q_1}$ = 2.45, the minimum occurs at (-0.0528,-266.55) and the maximum occurs at (-0.0495, 86.235). When ${q_2}/{q_1}$ increases from 2.451 to 10, there is an increase in Nusselt number as Br_{q_1} , increases. When ${q_2}/{q_1}$ = 2.451, the non-asymptotic curve has the minimum at (-0.0528,-250.25) and the maximum at (-0.0494, 83.5). Therefore when ${q_2}/{q_1}$ increases from 2.45 to 2.451, the Nusselt number changes from decreasing

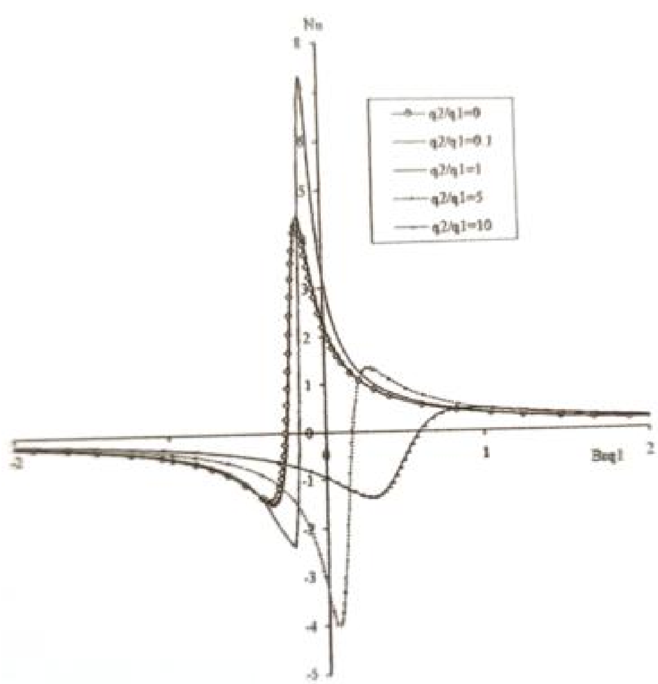

Fig. 3. Graph of Nu versus Br_{q_1} for $n = 2$

to increasing. It is evident that the heat transfer properties of a dilatant fluid with a power-law index depend significantly on the magnitude of the heat flux ratio and the Brinkman number.

Based on Eq. (34), four sets of curves are shown in Fig. 3, for the heat flux ratios of 0, 1, 5, and 10, for $n = 2$. It is observed again that the curves are not asymptotic and they have the maximum and minimum values for Nu. When q_2/q_1 = 0, 1, 5, 10, the minimum value that Nu takes is -1.4346, -2.4522, -3.9904, -1.3619, respectively, whereas the maximum value that Nu takes is 4.29, 7.3339, 1.3344, 0.2554 respectively.

It is noted that for pseudo-plastic fluids, when $n = 1/4$ and for Newtonian fluids when $n = 1$ the Nusselt number profiles against Br_{q_1}, are asymptotic, and for dilatant fluids, when $n = 2$ the

Nusselt number profiles against Br_{q_1}, are not asymptotic, but they have turning points as explained in Table 1.

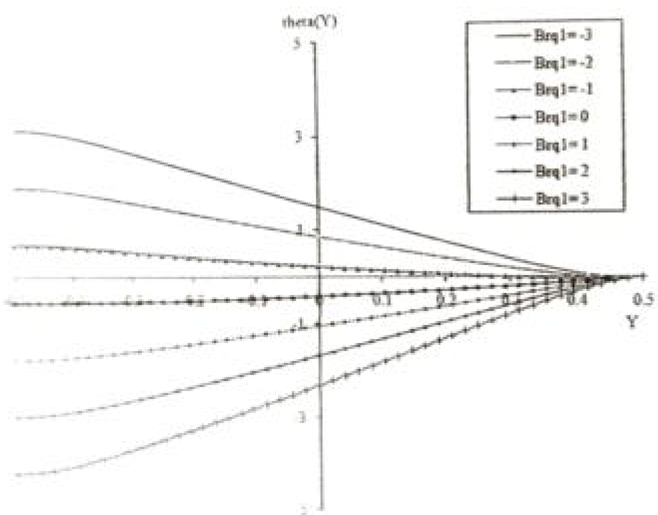

Fig. 4. Graph of $\theta(Y)$ versus Y for the case of insulated lower plate at $n = 1/4$.

3.2. Special Case of Insulated Lower Plate

For the case of insulated lower plate, $q_2 = 0$, and for Newtonian fluid, we obtain the established result

$$Nu = \frac{35}{13 + 12Br_{q_1}} \tag{35}$$

agreeing with Ref. [5].

For the pseudo-plastic fluids, from Fig. 4, for $n = 1/4$, it is observed that when Br_{q_1} = -3, -2, -1, the temperature distribution assumes positive values and it becomes 0 at Y = 0.5. When Br_{q_1} = 0, 1, 2 and 3, the temperature distribution assumes negative values and it becomes 0 at Y = 0.5.

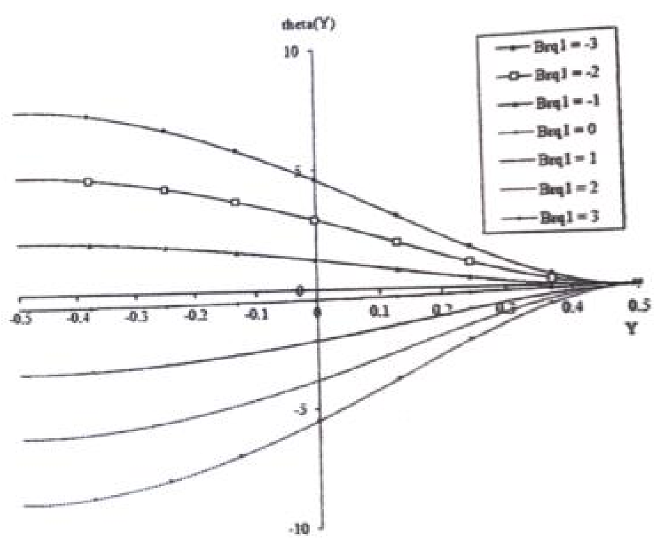

Fig. 5. Graph of $\theta(Y)$ vs Y for the case of insulated lower plate at $n = 2$.

For the dilatant fluids, from Fig. 5, for $n = 2$, the real part of theta is plotted. It is observed that when Br_{q_1} = -3, -2, -1, the temperature distribution assumes positive values and it becomes 0 at Y

= 0.5. When Br_{q_1} = 0, 1, 2 and 3, the temperature distribution assumes negative values and it becomes 0 at Y = 0.5. For $n = 1/4$,

$$Nu = \frac{1}{16/39 + \left(24711/33721\right) Br_{q_1}} \tag{36}$$

From Fig. 2, it is observed that when $q_2 = 0$ at $n = 1/4$, Nu versus Br_{q_1}, is asymptotic and the asymptote appears at Br_{q_1} = -0.55984. For $n = 1/2$,

$$Nu = \frac{1}{4/9 + \left(56638/63061\right) Br_{q_1}} \tag{37}$$

For dilatants, at $n = 2$, the real part of Nu is

$$Nu = \frac{1}{1102977 Br_{q_1}^2 + 463968 Br_{q_1} + 57028}, \tag{38}$$

verifying the findings in Table 1 and Fig. 3.

3.3. Equal Heat Fluxes Case

The particular interest here is the case when both the upper and lower plates are of equal heat flux, i.e., $q_1 = q_2$. An implicit expression was given in Bird et al., but our explicit form, in Eq. (25) with q_2/q_1 is ready to be used.

3.3.1. Newtonian Fluids

For the Newtonian fluid, the Nusselt number is reduced to

$$Nu = \frac{70}{17 + 24Br_{q_1}} = \frac{70}{17 + 27Br'_{q_1}} \tag{39}$$

where

$$Br'_{q_1} = \frac{\eta \bar{u}^{n+1}}{q_1 w^n}, \tag{40}$$

with $\bar{u}$ the mean velocity of the fluid. The expression of Nu in Eq. (39) corresponds to the classical problem of Poiseuille viscous-dissipative Newtonian flow in parallel plate channel. For verification of the present model, we observe that the $Nu - Br_{q_1}$ correlations in Eq. (39) are identical to those in [4] and [1], respectively, for fully developed flow of Newtonian fluid with isoflux boundary condition. For the case of no viscous dissipation, $Br_{q_1} = 0$, the Nusselt number becomes $Nu = 70/17$.

3.3.2. Shear Thinning Fluids

For the pseudo-plastic fluids, when $n = 1/4$,

$$Nu = \frac{1}{68/273 + \left(24711/33721\right) Br_{q_1}}, \tag{41}$$

and when $Br_{q_1} = 0$, $Nu = 273/68$.When $n = 1/2$,

$$Nu = \frac{1}{13/45 + \left(56638/63061\right) Br_{q_1}}, \tag{42}$$

and when $Br_{q_1} = 0$, Nu =45/13.

3.3.3. Shear Thickening Fluids

For dilatants, when $n = 2$, the real part of Nu is,

$$Nu = \frac{126\left(656 + 3753 Br_{q_1}\right)}{1102977 Br_{q_1}^2 + 321408 Br_{q_1} + 26233}, \tag{43}$$

and when $Br_{q_1} = 0$, Nu =82656/26233.

4. Conclusions

For fully developed power-law fluid flow between stationary parallel plates, an exact expression for the Nusselt number has been found. The investigation of heat transmission has revealed that the impact of viscous dissipation is crucial. The dimensionless temperature distribution and Nusselt number are obtained by Equations (11 and 25), respectively, when both plates are maintained at distinct constant heat fluxes, for all $n > 0$ and they are in terms of Br_{q_1}. When the lower plate insulated and the upper plate is at constant heat flux, the Nusselt number is derived by substituting $q_2 = 0$ in Eq. (25), and selected results are Eqs. (35)-(38). For the case of equal constant heat fluxes at both the plates, the Nusselt number is obtained by substituting $q_1 = q_2$ in Eq. (25), and selected results are Eqs. (39), (41)-(43). For $n = 1/4$, the Nusselt number distribution against Brq, is asymptotic, whereas, for $n = 2$, Nusselt number distribution against Br_{q_1}, is not asymptotic and maximum and minimum values occur at various points depending upon the ratio $\left(q_2 / q_1 \right)$.

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